



Authors' Reply to "Comments on "On a Misconception Involving

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Comments

Comments on "On a Misconception Involving Point Collocation and the Rayleigh Hypothesis"

Alexander B. Manenkov

In the paper cited above,¹ the relation between the Rayleigh hypothesis and convergence of the collocation methods is discussed. Investigating the problem of wave scattering by a sinusoidal grating, the authors drew a conclusion that "success or failure of the simple point-collocation method is *not* related to whether or not the Rayleigh hypothesis is satisfied" (see Section V of the paper¹). Quoting our publications (points 24–26 in the reference list of the paper¹) they inaccurately interpret the results obtained. In this comment I should like to call the readers' attention to the fact that the inferences, based on the rigorous interpolation theory, conflict with the conclusions of the paper.¹

In [1] and [2] we examined the application of the collocation technique to the problem of waves diffraction by different scatterers (the metallic cylinders, gratings, etc.), when the Rayleigh series are used. We looked for the point (node) distributions, which ensure the convergence of the collocation algorithms for the scatterers of sufficiently arbitrary forms. The distributions are demonstrated to be the same as for the problem of interpolation of analytical functions by polynomials in complex domains. Under condition $k \rightarrow 0$ (k is the wavenumber), the above connection of both problems can be easily explained, since the scattering problem is close to the static one. As following from this well-developed theory (see, for example, [3], [4]) there are acceptable nodes distributions, ensuring the numerical process convergence. In turn, the structure of these distributions depends on the singularities of the analytical function under examination outside the domain considered. Specifically, if these singularities are placed far from the domain boundary, the set of the acceptable node distributions are wide.

The same results are valid for the diffraction problems [1], [2]. In this case, the singularities of the fields are defined by the form of the scatterer boundary and the structure of the incident wave. To describe the results obtained, introduce a parameter ξ of the problem in question. For the problem of the plane wave scattering by the sinusoidal grating the parameter $\xi = \kappa b$, if we denote the grating surface by $y = b \cos \kappa x$. Let ξ_m is a "critical" value: for $\xi > \xi_m$ the Rayleigh hypothesis is invalid and vice versa. When $\xi < \xi_m$ there is a wide set of node distributions, ensuring convergence of the collocation method in question. For example, if $\xi = \kappa b = 0$ (for the plane surface) the true results are obtained practically for an arbitrary positioning of the collocation points along the plane [4]. However, if $\xi \gg \xi_m$, only the Fejer and similar nodes [2]–[4] ensure the convergence of the method. For the concrete geometry of a scattering

problem the set of acceptable point distributions can be theoretically found by the methods described in [3] and [4]. Of course, the practical algorithms of such calculations can be intricate.

Thus, the validity/invalidity of the Rayleigh hypothesis places limitations on the *acceptable* positioning of the nodes [1], [2]. For example, there are such node distributions that give the exact results *only*, if the Rayleigh hypothesis is valid and they produce incorrect data in opposite cases. This inference conflicts with the results of the paper. Our conclusion was verified by the numerical results, presented in other papers of the author [5], [6]. Small numbers of the examples considered in the paper cited above¹ do not allow to find the above set of the nodes positioning (for example, the Fejer distributions). It seems their interesting technique can be used to search such distributions.

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Authors' Reply

Søren Christiansen and Ralph E. Kleinman

Prof. Manenkov has put forward some illuminating comments¹ to our paper², where we have discussed the relation between the Rayleigh hypothesis and convergence of the collocation method. For the sinusoidal grating $y = b \cos \kappa x$, the parameter $\xi = \kappa b$ is introduced with ξ_m being a "critical" value (≈ 0.448). For $\xi > \xi_m$,

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¹A. B. Manenkov, "Comments on 'On a misconception involving point collocation and the Rayleigh hypothesis,'" *IEEE Trans. Antennas Propagat.*, this issue, p. 1764.

²S. Christiansen and R. E. Kleinman, "On a misconception involving point collocation and the Rayleigh hypothesis," *IEEE Trans. Antennas Propagat.*, vol. 44, pp. 1309–1316, Oct. 1996.

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¹S. Christiansen and R. E. Kleinman, *IEEE Trans. Antennas Propagat.*, vol. 44, pp. 1309–1316, Oct. 1996.

the Rayleigh hypothesis is invalid and for $\xi < \xi_m$, the Rayleigh hypothesis is valid. In our paper,² we tried to point out that when $\xi < \xi_m$, point collocation appears to converge for various node distributions but appears to diverge for some distributions, whereas when $\xi > \xi_m$, point collocation appears to diverge for many node distributions but appears to converge for some distributions.

We have given examples with nodes, which seem to produce divergence, but we do not claim that for all $\xi < \xi_m$ nodes can be positioned so that divergence appears; in particular, not for $\xi = 0$, which is a very special case. Although "...there is a wide set of node distributions, ensuring convergence...",¹ distributions giving divergence can be found for some values of ξ .

We have given examples with nodes, which seem to produce convergence, but we do not claim that for all $\xi > \xi_m$ nodes can be positioned so that convergence can be obtained; in particular, not for $\xi \gg \xi_m$, where the nodes for convergence should be chosen in a certain way.¹ The statement in the paper¹ is as follows: "Thus, the validity/invalidity of the Rayleigh hypothesis places limitations on the acceptable positioning of the nodes..." seems to us to be a supplement to our findings and we do not see that the statement is in any essential conflict with our results. On the other hand, we find that the profound investigations by Manenkov *et al.* support and clarify our findings.

Comments on "On the Use of ρ -Algorithm in Series Acceleration"

S. H. Tan

Although the authors of the abovementioned paper¹ have correctly stated that the ε algorithm is effective for accelerating the convergence of alternating series whereas the ρ algorithm is suitable for monotonic series, they have failed to recognize that sequences containing the spatial- or spectral-domain Green's function series partial sums are not monotonic, but oscillatory about certain limiting values. In fact, this suggests that the ε algorithm should be a more suitable accelerator for the spatial- or spectral-domain Green's function series. The use of an example of the monotonic series $S_n = \sum_{m=1}^n \frac{1}{m^2}$ to illustrate the effectiveness of the ρ algorithm over the ε algorithm is, therefore, not relevant in a paper whose objective concerns the study of effective accelerators for convergence of the spatial- or spectral-domain Green's function series.

To demonstrate our point, we have chosen the spectral-domain Green's function series studied in the paper¹

$$\begin{aligned} G(x-x', y-y') &= \sum_{m=-\infty}^{\infty} \frac{1}{2dq_0} e^{-q_0|x-x'|} e^{-j\frac{2\pi m(y-y')}{d}} \\ &= \sum_{m=0}^{\infty} \frac{\delta_m}{2dq_0} e^{-q_0|x-x'|} \cos\left(\frac{2\pi m(y-y')}{d}\right) \end{aligned} \quad (1)$$

where d is the separation between adjacent sources in the linear array of periodic sources along the y axis in a two-dimensional plane x -

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The author is with Nanyang Technological University, Singapore 639798. Publisher Item Identifier S 0018-926X(98)08908-X.

¹S. Singh and R. Singh, *IEEE Trans. Antennas Propagat.*, vol. 39, pp. 1514-1517, Oct. 1991.

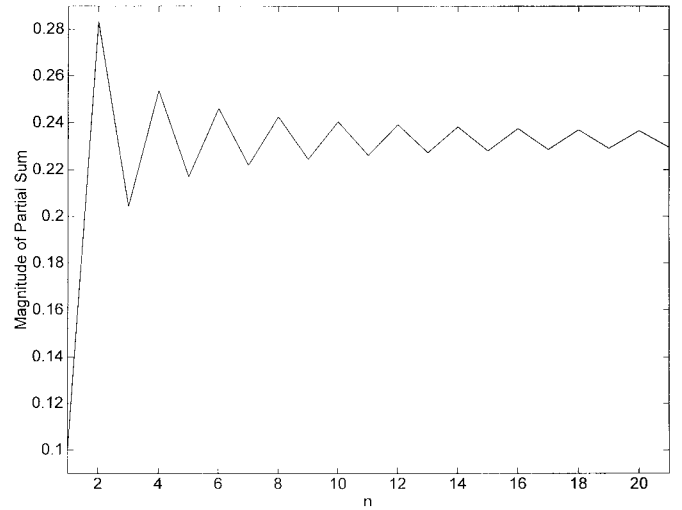


Fig. 1. Plot of magnitude of partial sum $|S_n|$ versus number of terms n .

TABLE I
COMPARISON OF SERIES SUMMATION FOR $\text{Re}[G]$.
 $\text{Im}[G] = 0.09947183943243$ IN ALL CASES

Method	No. of terms n	Approximation to $\text{Re}[G]$
Direct Sum	15,278,875	0.210612286275010
Direct Sum	152,788,746	0.210612280545785
ε -algorithm	17	0.21061228106648
ε -algorithm	21	0.21061228106667
ρ -algorithm	300	0.21155747352763
ρ -algorithm	400	0.21133193369144

y , (x', y') denotes the source coordinates, and (x, y) denotes the observation coordinates $q_0 = \sqrt{(\frac{2m\pi}{d})^2 - k_0^2}$, $k_0 = \frac{2\pi}{\lambda_0}$, $\delta_m = 1$ if $m = 0$ and $\delta_m = 2$ if $m > 0$. For the case of $x = x'$, convergence of the series for $y \neq y'$ is generally quite slow for the spectral-domain series. In the limit as $y \rightarrow y'$, the series is divergent, giving rise to the familiar Green's function singularity as the observation point approaches the source point.

For numerical computation we have chosen $\lambda_0 = 1.2$ m, $d = 0.8\lambda_0$, $y - y' = 0.4\lambda_0$, $x - x' = 0.0$ (corresponding to the data set used in Fig. 5¹). The direct sum S is first computed to a desired accuracy from (1) using the double precision mode of FORTRAN 77 on a high-performance DEC ALPHA 8400 machine. Using MATLAB on a pc, the n -term partial sum sequence $S_0, S_1, S_2, \dots, S_n$ for a suitable n is generated and the ε algorithm or the ρ algorithm is applied to the sequence to find S and the approximation to G . A plot of the partial sum S_n versus n is shown in Fig. 1. The results of the numerical experiment are summarized in Table I.

From the results above, it is obvious that the ρ algorithm is hardly suitable for the spectral-domain Green's function G , which exhibits an oscillatory partial sum value about its limiting value. In fact, the conclusion should be the opposite of that reached in the paper,¹ namely, the ε algorithm is more effective for the type of Green's function series studied in the paper.¹